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LARGE DEFLECTION OF RECTANGULAR SANDWICH PLATES

JAO-SHIUN KAO

Department of Civil Engineering, Marquette University, Milwaukee, Wisconsin 53233, U.S.A.

LUIS GUZMAN-BARRON

Division of Engineering, Catholic University, Lima, Peru

and

ALOIS J. HARTMANN

Department of Civil Engineering, Marquette University, Milwaukee, Wisconsin 53233, U.S.A.

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Abstract—The governing differential equations and the boundary conditions for the large deflection of rectangular sandwich plates are derived using the principle of the complementary energy. The governing differential equations are transformed into systems of nonlinear algebraic equations using the finite difference method, and solved by successive iteration. For the purpose of illustration, deflection behavior of simply supported rectangular plates under uniform load is presented. The deflection behavior of plates with various values of shear rigidities and intensity of applied loads is studied. The change in the stress patterns of the face layers of the plate is also discussed.

NOTATION

a, b	dimension of the plate
h	plate thickness, distance between the centers of two face layers (Fig. 2)
q	$\dfrac{\bar{q}a^4}{16Eth^3}$, non-dimensional load
$\bar{q}$	uniform transverse load
t	thickness of the face layers
u	deformation in the x-axis direction
v	deformation in the y-axis direction
w	deformation in the z-axis direction (deflection)
w_b	deflection due to bending only
w_s	deflection due to shear only
D	$\dfrac{Eth^2}{2(1-\nu^2)}$, flexural rigidity of sandwich plates
E	elastic modulus of face layers
$\bar{F}$	assumed stress function at the boundary
G_c	shear modulus of the core
M_x, M_y	bending moments in the x- and y-directions, respectively
N_x, N_y, N_{xy}	membrane forces in transverse loaded plate
$N_1, N_2, N_3,$	membrane forces in plane stress problem
Q_x, Q_y	shearing forces
S	$G_c h$
α_b	coefficient for deflection due to bending, equation (52)
α_s	coefficient for deflection due to shear, equation (53)

β	slope in x direction for the sandwich plate, equation (14)
γ	slope in y direction for the sandwich plate, equation (15)
γ_{xy}, γ_{yx}	shearing strains
ρ	$\dfrac{G_c}{E} \times 10^3$
ν	Poisson's ratio
∇^2	$\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$
ϕ	function defined by equation (22).
ψ	function defined by equation (23).

1. INTRODUCTION

Due to the extensive use of sandwich type construction in the fabrication of major structural components, the bending and buckling behavior of sandwich plates have been widely investigated[1–4]. However, most of these investigations are based on the small deflection theory. The small deflection theory may give satisfactory results if the deflections are small compared to the thickness of the plate. However, if the deflections are of the same order as the thickness of the plate, the stretching of the middle surface should be taken into account, i.e. the large deflection theory should be used. The governing equations for the large deflection of sandwich plates were first derived by Reissner[5]. Wang[6] later obtained the equations for the large deflection of thin homogeneous and sandwich plates and shells using the principle of complementary energy. Due to the complexity of the non-linear equations no practical problem was studied by either author.[3, 4].

Large deflection behavior of rectangular sandwich plates has been investigated by Kan and Huang[7] and Awan[8]. The first of these two papers used a perturbation method and only the deflection of rectangular sandwich plates with all boundaries fixed was considered. Awan[8] used a series solution to solve the two governing differential equations derived by Reissner[5] for the deflection of simply supported rectangular sandwich plates with all boundaries restrained against in-plane movement. This type of boundary condition, as pointed out by Folie[9], is very rare in practice. A deficiency of the method presented in these papers[7, 8], was the inability to obtain the stresses in the plate which are more important in the practical design than the deflection[9].

In this paper, in addition to the two differential equations obtained by Reissner[5], the third differential equation which is needed for the evaluation of the stresses in the plates and the boundary conditions for the large deflection of sandwich plates are derived. Expressions for stress resultants are also obtained in terms of deflection, a stress function and auxiliary functions. The three governing partial differential equations are then transformed into systems of nonlinear algebraic equations using the finite difference method, and solved by successive iteration. Foı the purpose of illustration, simply supported rectangular plates under uniform load are considered. The edges of the plates are assumed to be free to move in the direction of the plane of the plate (no restriction is imposed on the rigid movement of the edges). The convergence criteria are discussed. A comparison with the deformations in small deflection theory is made using the results given by Plantema[2], and then the method is applied to the study of the large deflection behavior of sandwich plates with various types of shear rigidities and applied load intensities. The change in the stress patterns in the plate is also discussed. The assumptions made with regards to the sandwich plate are: (1) The face layers are membranes and they are of same material and of equal thickness; (2) both the core and face layers are isotropic; and (3) the core takes only the transverse shear stress.

2. GOVERNING EQUATIONS

Consider a rectangular sandwich plate of dimensions a by b with two facings of thickness t. Figure 1 shows the coordinate axis used and Fig. 2 shows an element of plate. The first two governing differential equations have been derived by Reissner[5]. They are

$$\nabla^2\nabla^2 F = 2Et\left[\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2}\right] \tag{1}$$

and

$$D\nabla^2\nabla^2 w = \left[1 - \frac{D}{hG_c}\nabla^2\right]\left[\bar{q} + \frac{\partial^2 F}{\partial y^2}\cdot\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x\,\partial y}\cdot\frac{\partial^2 w}{\partial x\,\partial y} + \frac{\partial^2 F}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2}\right]. \tag{2}$$

In order to obtain the third governing differential equation the relations between the stress resultants and displacements are needed. These conditions together with the boundary conditions may be obtained by taking the first variation of the complementary energy using the equilibrium equations as constraint conditions[6, 10–12].

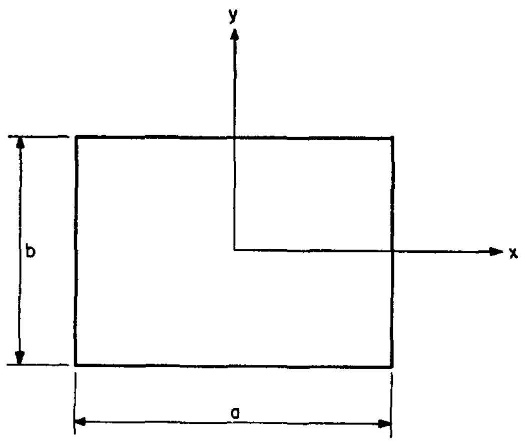

Fig. 1. Rectangular sandwich plate.

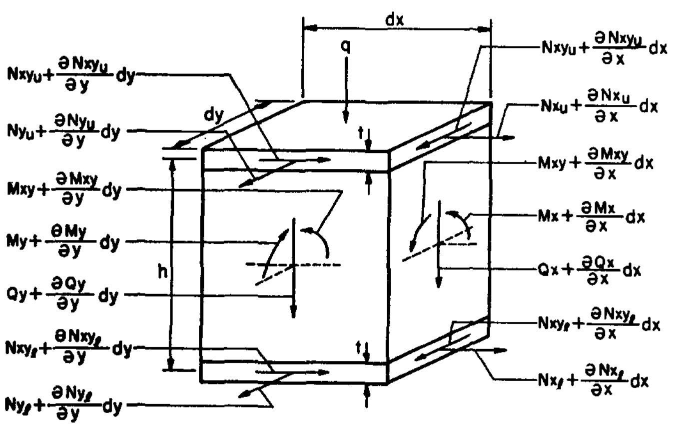

Fig. 2. Element of sandwich plate.

The equilibrium equations are

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \tag{3}$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0 \tag{4}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + \bar{q} + N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy} \frac{\partial^2 w}{\partial x\, \partial y} + N_y \frac{\partial^2 w}{\partial y^2} = 0 \tag{5}$$

$$\frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} - Q_x = 0 \tag{6}$$

$$\frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} - Q_y = 0. \tag{7}$$

The stress-resultants and displacement relations are

$$\frac{N_x - \nu N_y}{2Et} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{8}$$

$$\frac{N_y - \nu N_x}{2Et} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \tag{9}$$

$$\frac{(1 + \nu)N_{xy}}{Et} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial y} \cdot \frac{\partial w}{\partial x} \tag{10}$$

$$M_x - \nu M_y = D(1 - \nu^2) \frac{\partial \beta}{\partial x} \tag{11}$$

$$M_y - \nu M_x = D(1 - \nu^2) \frac{\partial \gamma}{\partial y} \tag{12}$$

$$M_{xy} = -D \frac{1 - \nu}{2} \left(\frac{\partial \beta}{\partial y} + \frac{\partial \gamma}{\partial x}\right) \tag{13}$$

$$\beta = -\frac{\partial w}{\partial x} + \frac{Q_x}{G_c h} \tag{14}$$

$$\gamma = -\frac{\partial w}{\partial y} + \frac{Q_y}{G_c h} \tag{15}$$

and the boundary conditions are:
(a) For simply supported edges
at $x = 0$ and a

$$N_x = N_1 \quad \text{or} \quad u = 0 \tag{16a}$$

$$N_{xy} = N_3 \quad \text{or} \quad v = 0 \tag{16b}$$

$$w = 0, \qquad M_x = 0, \qquad M_{xy} = 0 \text{ (or } \gamma = 0) \tag{16c}$$

at $y = 0$ and b

$$N_y = N_2 \quad \text{or} \quad v = 0 \tag{17a}$$

$$N_{yx} = N_3 \quad \text{or} \quad u = 0 \tag{17b}$$

$$w = 0, \qquad M_y = 0, \qquad M_{yx} = 0 \text{ (or } \beta = 0\text{)}. \tag{17c}$$

(b) For fixed edges
at $x = 0$ and a

$$N_x = N_1 \quad \text{or} \quad u = 0 \tag{18a}$$

$$N_{xy} = N_3 \quad \text{or} \quad v = 0 \tag{18b}$$

at $y = 0$ and b

$$N_y = N_2 \quad \text{or} \quad v = 0 \tag{19a}$$

$$N_{yx} = N_3 \quad \text{or} \quad u = 0 \tag{19b}$$

$$w = 0, \qquad \gamma = 0, \qquad \beta = 0. \tag{19c}$$

(c) For free edges
at $x = 0$ and a

$$N_x = N_1 \tag{20a}$$

$$N_{xy} = N_3 \quad \text{or} \quad u = 0 \tag{20b}$$

$$M_x = 0, \qquad M_{xy} = 0 \text{ (or } \gamma = 0\text{)}, \qquad N_x\left(\frac{\partial w}{\partial y}\right) + N_{xy}\left(\frac{\partial w}{\partial y}\right) + Q_x = 0 \tag{20c}$$

at $y = 0$ and b

$$N_y = N_2 \tag{21a}$$

$$N_{yx} = N_3 \quad \text{or} \quad u = 0 \tag{21b}$$

$$M_y = 0, \qquad M_{yx} = 0 \text{ (or } \beta = 0\text{)}, \qquad N_y\left(\frac{\partial w}{\partial y}\right) + N_{yx}\left(\frac{\partial w}{\partial x}\right) + Q_y = 0. \tag{21c}$$

It should be noted that the conditions, $M_{xy} = 0$ or $\gamma = 0$ in equations (16c) and (20c), and $M_{yx} = 0$ or $\beta = 0$ in equations (17c) and (21c) depend on the edge condition of the plate. If the shear strains are prevented by the presence of an edge stiffener, the condition $\gamma = 0$ (or $\beta = 0$) should be used. If no edge stiffener is applied to prevent shear strains, then $M_{xy} = 0$ (or $M_{yx} = 0$) should be used.

Introduce two new functions such that

$$\phi = \frac{\partial \beta}{\partial x} + \frac{\partial \gamma}{\partial y} \tag{22}$$

$$\psi = \frac{\partial \beta}{\partial y} - \frac{\partial \gamma}{\partial x}. \tag{23}$$

Substituting equations (14 and 15) into equation (22) with the aid of equation (5) and noting that

$$N_x = \frac{\partial^2 F}{\partial y^2} \tag{24}$$

$$N_y = \frac{\partial^2 F}{\partial x^2} \tag{25}$$

$$N_{xy} = -\frac{\partial^2 F}{\partial x\,\partial y} \tag{26}$$

yields

$$\phi = -\nabla^2 w - \frac{1}{G_c h}\left[\bar{q} + \frac{\partial^2 F}{\partial y^2}\cdot\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x\,\partial y}\cdot\frac{\partial^2 w}{\partial x\,\partial y} + \frac{\partial^2 F}{\partial x^2}\cdot\frac{\partial^2 w}{\partial y^2}\right]. \tag{27}$$

Solving for stress resultants from equations (11–15), yields

$$M_x = D\left[\frac{\partial \beta}{\partial x} + \nu\frac{\partial \gamma}{\partial y}\right] \tag{28}$$

$$M_y = D\left[\nu\frac{\partial \beta}{\partial x} + \frac{\partial \gamma}{\partial y}\right] \tag{29}$$

$$M_{xy} = -\frac{D}{2}(1-\nu)\left[\frac{\partial \beta}{\partial y} + \frac{\partial \gamma}{\partial x}\right] \tag{30}$$

$$Q_x = G_c h\left[\beta + \frac{\partial w}{\partial x}\right] \tag{31}$$

$$Q_y = G_c h\left[\gamma + \frac{\partial w}{\partial y}\right]. \tag{32}$$

Substituting equations (28–32) in equations (6 and 7), with the aid of equations (22 and 23) and then solving for β and γ yields, respectively, to

$$\beta = \frac{D}{G_c h}\left[\frac{\partial \phi}{\partial x} + \frac{1-\nu}{2}\frac{\partial \psi}{\partial y}\right] - \frac{\partial w}{\partial x} \tag{33}$$

$$\gamma = \frac{D}{G_c h}\left[\frac{\partial \phi}{\partial y} - \frac{1-\nu}{2}\frac{\partial \psi}{\partial x}\right] - \frac{\partial w}{\partial y}. \tag{34}$$

Introducing β and γ in equations (23) and simplifying yields

$$\nabla^2\psi - \frac{2hG_c}{D(1-\nu)}\psi = 0 \tag{35}$$

which is the third governing differential equation. The same equation was obtained by Kao[11]. This equation is in the same form as the one obtained by Reissner[12] for homogeneous plates with shear deformation.

The expressions for the stress resultants in terms of ϕ, ψ and w can be obtained from equations (28–32) with the aid of equations (33 and 34). Thus

$$M_x = D\left\{\frac{D}{G_c h}\left[\frac{\partial^2\phi}{\partial x^2} + \nu\frac{\partial^2\phi}{\partial y^2} + \frac{(1-\nu)^2}{2}\frac{\partial^2\psi}{\partial x\,\partial y}\right] - \left[\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right]\right\} \tag{36}$$

$$M_y = D\left\{\frac{D}{G_c h}\left[\nu\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - \frac{(1-\nu)^2}{2}\frac{\partial^2\psi}{\partial x\,\partial y}\right] - \left[\nu\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right]\right\} \tag{37}$$

$$M_{xy} = -\frac{D(1-\nu)}{2}\left\{\frac{D}{hGc}\left[2\frac{\partial^2\phi}{\partial x\,\partial y} - \frac{1-\nu}{2}\left(\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2}\right)\right] - 2\frac{\partial^2 w}{\partial x\,\partial y}\right\} \tag{38}$$

$$Q_x = D\left[\frac{\partial\phi}{\partial x} + \frac{1-\nu}{2}\frac{\partial\psi}{\partial y}\right] \tag{39}$$

$$Q_y = D\left[\frac{\partial\phi}{\partial y} - \frac{1-\nu}{2}\frac{\partial\psi}{\partial x}\right]. \tag{40}$$

3. METHOD OF SOLUTION

The three governing partial differential equations, equations (1, 2 and 35) are transformed into systems of algebraic equations using finite differences, and solved by successive iterations. For the purpose of illustration, a simply supported rectangular plate with edge stiffener under uniform load is considered herewith. The boundary conditions for the deflection and stress resultants in this case can be readily shown from equations (16, 17, 33–36 and 37) to be

$$w = 0 \tag{41}$$

$$\frac{\partial^2 w}{\partial x^2} = \frac{D}{hG_c}\left[\frac{\partial^2\phi}{\partial x^2} + \nu\frac{\partial^2\phi}{\partial y^2} + \frac{(1-\nu)^2}{2}\frac{\partial^2\psi}{\partial x\,\partial y}\right] \tag{42}$$

$$\frac{\partial\psi}{\partial x} = \frac{2}{(1-\nu)}\frac{\partial\phi}{\partial y} \tag{43}$$

along $x = 0$ and $x = a$, and

$$w = 0 \tag{44}$$

$$\frac{\partial^2 w}{\partial y^2} = \frac{D}{hG_c}\left[\nu\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} - \frac{(1-\nu)^2}{2}\frac{\partial^2\psi}{\partial x\,\partial y}\right] \tag{45}$$

$$\frac{\partial\psi}{\partial y} = -\frac{2}{1-\nu}\frac{\partial\phi}{\partial x} \tag{46}$$

along $y = 0$ and $y = b$.

The boundary conditions for in-plane force can be converted to geometric constraints on F, for constant edge normal forces, by assuming a stress function that satisfies the equilibrium equations near the edges. Equations (3) and (4) are satisfied at the boundaries if the stress function F is assumed to be $\bar{F}$, where $\bar{F}$ is the solution of the plane stress problem corresponding to the edge loading on the actual plate. For the case of constant edge forces, $\bar{F}$ is given by

$$\bar{F} = \frac{N_1}{2}(x^2 - ax) + \frac{N_2}{2}(y^2 - by). \tag{47}$$

Thus in view of equations (24–26) the edge constraints on F become

$$\frac{\partial^2 F}{\partial x^2} = \frac{\partial^2 \bar{F}}{\partial x^2} = -\frac{N_1 a}{2} = N_y \quad \text{along} \quad y = 0 \quad \text{and} \quad y = b \tag{48}$$

$$\frac{\partial^2 F}{\partial y^2} = \frac{\partial^2 F}{\partial y^2} = -\frac{N_2 b}{2} = N_x \quad \text{along} \quad x = 0 \quad \text{and} \quad x = a. \tag{49}$$

The conditions imposed on function ψ are that ψ is antisymmetric function, i.e.

$$\psi = \text{o} \quad \text{along} \quad x = \frac{a}{2} \quad \text{and} \quad y = \frac{b}{2} \tag{50}$$

and at the edges ψ is given by the boundary conditions on γ_x and β_y for plates with edge stiffeners and on M_{xy} for plates without edge stiffeners.

With the boundary conditions for w, F and ψ specified and the equations (1, 2 and 35) in the finite difference form, the first iteration assumes that ψ is zero and F is zero or equal to an initial plane stress condition and equation (2) is solved for w. This result will correspond to the small deflection theory without shear deformation. The procedure for the next iteration is: (1) Use the previous values of w and assume that the plate has that shape, to evaluate the derivatives of w. A similar procedure is used for F (which would be zero or constant for the first time). Knowing the derivatives of w and F, the auxiliary function ϕ, can be evaluated from equation (27) in which the constant, $\bar{q}$, is neglected since only the derivatives of ϕ are needed. (2) Evaluate the derivatives of ϕ to set up the boundary conditions for ψ, and then solve equation (35) for ψ. (3) Assuming ϕ, ψ, and w, and their derivatives are known from previous cycle, solve equation (1) for F. (4) Finally, with ϕ, ψ, and F as constants, find a new set of values for the deflection w. Compare the new values of w with the previous ones and if they do not agree within a prescribed tolerance, repeat steps 1, 2, 3, and 4 until they do.

4. CONVERGENCE CRITERIA

The computer program is set to store the relative difference between 3 consecutive values of w at the center of the plate, namely

$$\text{check } 3 = (w_i - w_{i-1})/w_i$$
$$\text{check } 2 = (w_{i+1} - w_i)/w_{i+1}.$$

Whenever both differences are less than or equal to 0·004, it is assumed that an acceptable convergence has been achieved.

The reason for this double check in convergence is due to the fact that successive values of w are of an oscillatory nature, and with this test it may be safely assumed that the point taken as the real deflection is not in the highest (or lowest) part of one of the oscillations, unless the oscillation itself is already very small.

From Fig. 3 it is possible to find the values w_{i-1}, w_i and w_{i+1}, corresponding to three successive iterations. If the convergence criteria were based on the difference between two consecutive values of w only, then the final answer would be taken as w_i or w_{i+1}, which might differ 10 or 15 per cent from the real answer.

If a third value is added, the convergence criteria will not give w_i or w_{i+1} as an answer, unless the difference between w_{i+1} and w_{i-1} is also very small.

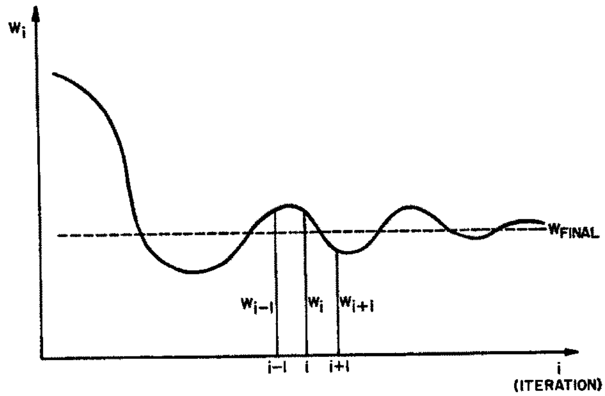

Fig. 3. Convergence of the solution.

5. COMPARISON WITH PREVIOUS RESULTS—SMALL DEFLECTION THEORY

The approach employed herein is first used to analyze the small deflection behavior of sandwich plates. The results obtained are compared with those given by Plantema[2] who assumed that the shear and bending deflections are not inter-related and thus can be superimposed. He introduces the "partial deflections" by putting

$$w = w_b + w_s \tag{51}$$

and

$$w_b = \alpha_b \frac{q_a^{\,4}}{D} \tag{52}$$

$$w_s = \alpha_s \frac{q_a^{\,4}}{S} \tag{53}$$

where w is the total deflection and w_b and w_s are the deflections due to bending and shear, respectively. Values of α_b and α_s are given by Plantema[2] for several aspect ratios. For a square plate, $\alpha_b = 0{\cdot}00406$ and $\alpha_s = 0{\cdot}0737$; in both cases the solution has been obtained using a series approximation.

Figure 4 shows the finite difference solution for various grid spacings. The results show that the calculated deflection increases as the number of grid points increase and that Platema's approach provides an upper bound. It should be mentioned here that the finite difference results will not converge to Plantema's because, from Plantema's solution

$$Q_x = S\frac{\partial w_s}{\partial x}$$

$$Q_y = S\frac{\partial w_s}{\partial y}$$

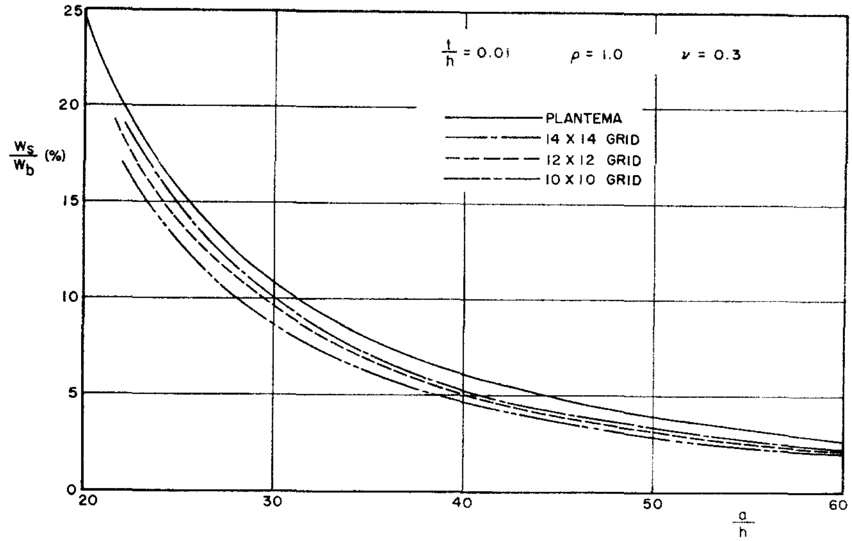

Fig. 4. Effect of number grid points on calculated shear deflection of square sandwich plate.

whereas, in the finite difference approach

$$Q_x = S\left(\beta + \frac{\partial w}{\partial x}\right)$$

$$Q_y = S\left(\gamma + \frac{\partial w}{\partial y}\right)$$

in order to match both solutions, it would be required that

$$\beta = -\frac{\partial w_b}{\partial x}$$

$$\gamma = -\frac{\partial w_b}{\partial y}$$

but β and γ actually depend on w, ϕ and ψ as shown in equations (33 and 34).

6. NUMERICAL RESULTS

The theory derived above has been used to study the effect of shear and its interaction with the plate membrane forces on the deformation behavior of a simply supported sandwich plate having infinitely rigid edge stiffeners; the plate is subject to a uniformly distributed transverse load. The effects studied are: (1) the load-deformation relationship, (2) the membrane force and bending moment distributions in the plate, and (3) the distribution of normal stresses in the top and bottom face layers. The results discussed below have been obtained for a sandwich plate having a face layer thickness to plate thickness ratio, t/h, equal to 0·01 and a span to plate thickness ratio, a/h, equal to 44·5.

The effect of core shear rigidity on the deformation behavior of a sandwich plate is shown in Figs. 5 and 6 in which the deflection has been normalized with respect to the plate thickness, h, and load is given in terms of the non-dimensional parameter, q. The dashed curve

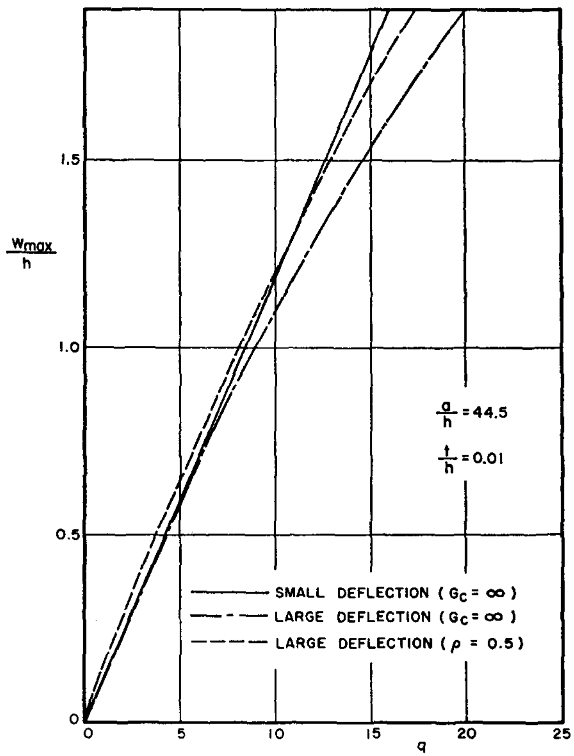

Fig. 5. Normalized deflection at the center of a square sandwich plate.

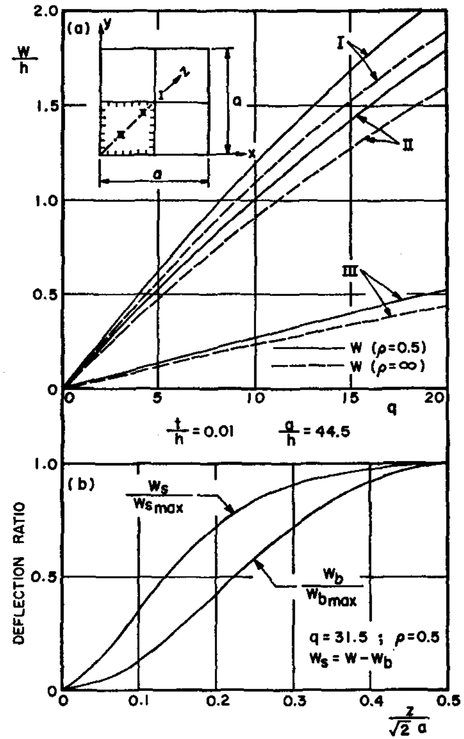

Fig. 6. Deflection along $x = y$ axis.

in Fig. 5 shows the effect of the interaction of the shear deformation and the plate membrane forces on the center deflection of the plate; the other two curves show the large and small deflection behavior of the plate where the core shear rigidity is infinite. A comparison of the dashed and solid curves (small deflection theory) indicates that the presence of shear deformation increases the load range over which the plate deformation is essentially linear. For the plate configuration studied, the shear deflection component varied from 8·3 per cent of the total deflection for q less than 4 to about 11·5 per cent for q equal to 15. Besides increasing the center deflection of the plate, shear also influences the deflected shape of the whole plate as shown in Figs. 6a and b. Figure 6a shows the effect of shear on the plate deflection at various points on the plate diagonal. Although Fig. 6a indicates an increase in deflection throughout the plate, it does not show how this increase is distributed. This is shown in Fig. 6b where the shear deflection, w_s, and bending deflection, w_b, components along the plate diagonal are shown normalized by their respective maximum values; the curves shown are for q equal to 31·5. Whereas the bending deflection increases slowly to its maximum value at the center, the shear deflection reaches over 80 per cent of its maximum value in the outer quarter of the plate.

The bending moments, M_x and M_y, along the ξ–ξ axis of the sandwich plate are shown in Fig. 7; these moments have been normalized with respect to the M_x value at the center of

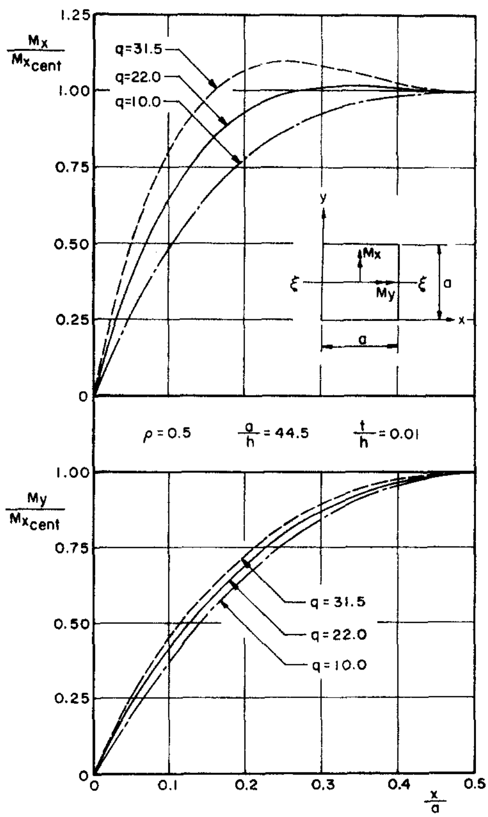

Fig. 7. Normalized bending moments along ξ–ξ axis.

the plate. The movement of the point of maximum bending moment, M_x, away from the center of the plate with increasing lateral load, q, is similar to that noted earlier[13, 14] for homogeneous plates; at $q = 31{\cdot}5$, the point of maximum moment has moved to approximately the quarter point on the center line of the plate. An analysis of the plate moments indicates that the presence of shear deformation increases the peak values of M_x and M_y by about 5 per cent over that obtained when the shear rigidity is infinite.

Figure 8 shows the variation of the membrane forces, N_x and N_y, along the ξ–ξ axis for various values of q; these forces have been normalized with respect to N_x at the center of the plate. Although the distribution of the membrane forces along the ξ–ξ axis of the plate is not significantly effected by the load magnitude, the maximum value of N_x at the center is increased by more than 400 per cent while the minimum value of N_y at the edge of the plate is increased by more than 500 per cent as q is increased from 10 to 31·5. In addition an analysis of the plate membrane forces indicates that the presence of shear deformation increases the maximum value by about 18 per cent and the minimum value of N_y by about 26 per cent over the corresponding values for the case where the core shear rigidity is infinite.

Finally Figs. 9 and 10 show the effect of load on the distribution of the total normal stress distribution in the x- and y-directions in the upper and lower face layers of the plate; Fig. 9 shows the variation of σ_x along the ξ–ξ axis of the plate in the x-direction. Although the maximum tensile stress occurs at the center of the plate, the maximum compressive stress occurs at a point away from the center; for q equal to 31·5 the maximum compressive

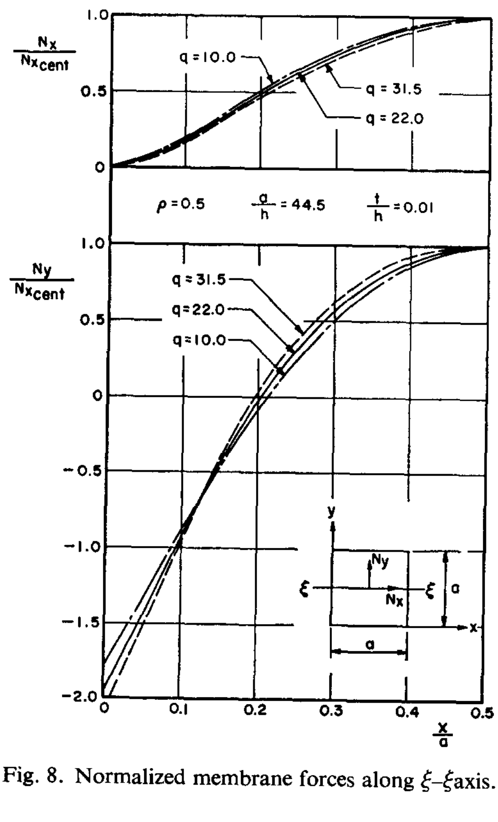

Fig. 8. Normalized membrane forces along ξ–ξaxis.

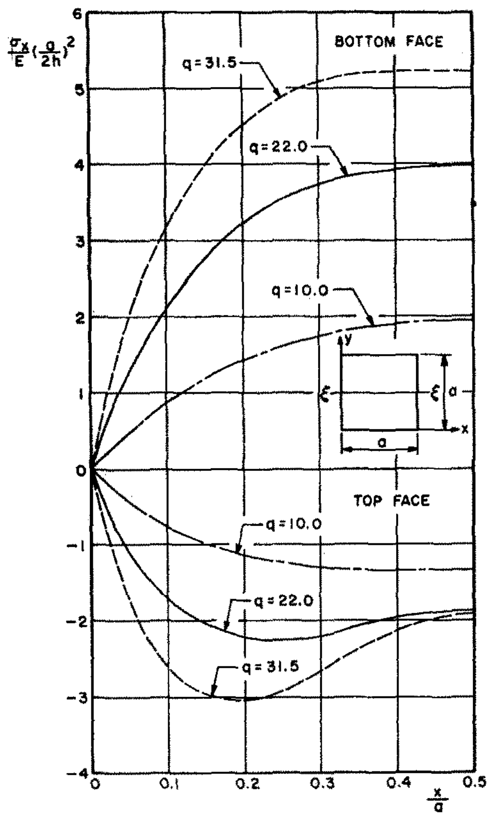

Fig. 9. Variation of normalized bending stresses in x direction along ξ–ξ axis.

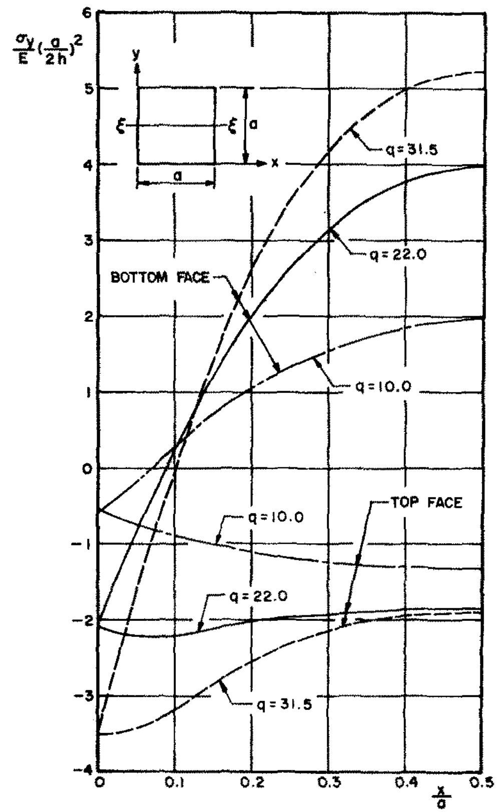

Fig. 10. Variation of normalized bending stresses in y direction along ξ–ξ axis.

stress is about 50 per cent greater than that at the center of the plate. Thus if the upper face layer were to fail by local buckling, it would occur away from the center of the plate. The variation of σ_y along the ξ–ξ axis is shown in Fig. 10 and it is notable that compressive stresses exist both in the top and bottom face layers.

7. CONCLUSIONS

The results of this study show that the method of analysis developed in this paper provides a useful tool for studying the large deflection behavior of sandwich plates. It should be emphasized that the finite difference formulation used, herein, makes it possible to obtain not only plate deflection but also the corresponding bending moments and membrane forces.

The method can also be extended to cover the case of shear deformation of homogeneous plates; this problem is currently under study.

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Абстракт—По принципу дополнительной энергии для большого прогиба прямоугольных плит с чередующимися слоями получили управляющие дифференциальные уравнения. Эти дифференциальные уравнения превратили методом финитной компенсации в системы нелинейных алгебраических уравнений, и их решили последовательным повторением. Представляется иллюстрация поведения прямоугольных плит, поддерживаемых обычным способом, при прогибе под однородной нагрузкой. Изучается поведение при прогибе плит с различными упругостями на сдвиг и с различными интенсивностями приложенной нагрузки. Также рассматриваются изменения в расположении деформационных напряженностей на поверхностных слоях плиты.